The Magic Picture Frame

You too can prove the

Theorem of Pythagoras

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Pythagoras' theorem states that for any right angled triangle with sides x,y,z, where z is the longest side or 'hypotenuse':

$$x^2 + y^2 = z^2$$

To introduce young children to Pythagoras, I recommend the geometric proof known hundreds of years before Euclid's more formal proof. The original was no doubt scratched on the desert sands but paper, scissors and coloured pens make the construction readily accessible.

Select any Pythagorean triangle with sides x,y,z – the 3,4,5 triangle is perhaps the easiest – and draw a square of side x+y on paper or card to make a "picture frame". As shown on the left of Figure 1, mark a distance y from the top left corner of the square, first along the top edge and then down the left side. From these two points draw a horizontal and vertical line to give two subsidiary squares, one of side x and one of side y; these new squares are therefore the "squares on the other two sides".

Draw a diagonal across each remaining rectangle so that there are four identical right-angled triangles of sides x,y,z. (Make sure the diagonals leave all four right-handed or all four left-handed.) Carefully cut out the original square leaving the picture frame and cut the square up into the two smaller squares and four identical triangles.

This 'proof by drawing' is for any x and y, not just counting numbers or integers. But there are some famous whole number triples known to wise men of yore: you might try such as 3,4,5. 3 and 4 give a hypotenuse of 5. Of course you will need suitable units to fit your paper Try 5,12 and 8,15or 20,21. What do you get for z?

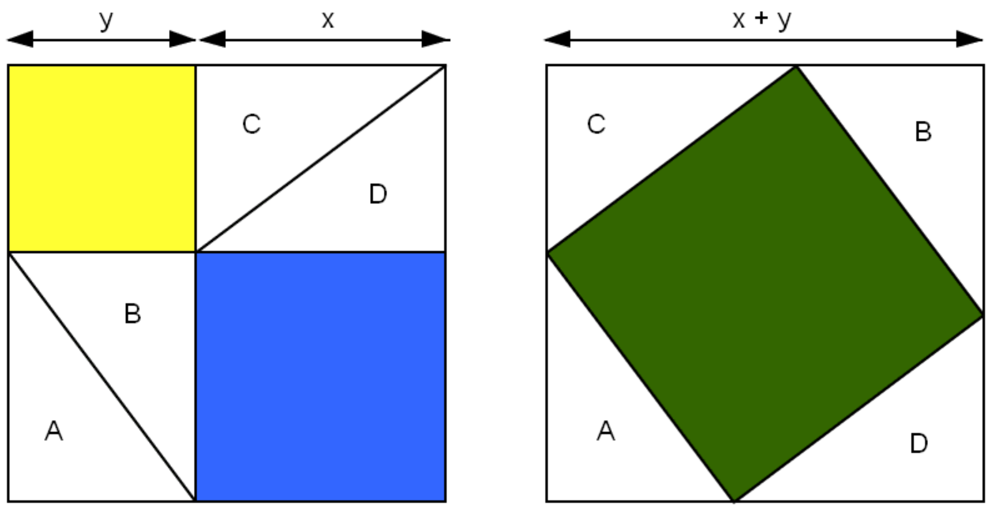

Figure 1: Constructing the "Magic Picture Frame" to prove Pythagoras' Theorem visually

Some decoration on the picture frame would be agreeable and you could mark the two squares with their area in the form $\begin{matrix} x \times x = \\ 3 \times 3 = 9 \end{matrix}$ $\begin{matrix} y \times y = \\ 4 \times 4 = 16 \end{matrix}$ say.

If the four triangles are returned to their original position, then the ‘cut out space’ is the two squares. But if the triangles are moved into symmetric positions in the magic picture frame as shown on the right of Figure 1, with their right-angles in the corners, then the cut out space must be the same in total area, but it has been transformed into a single square with the hypotenuse as side.

Place the empty picture frame, with the four triangles in the corners as shown on the right of Figure 1, over another sheet of paper and draw the tilted square hole in the middle onto the paper. Cut out the square you’ve just drawn, the square on the hypotenuse. Your three different squares can now be placed against the sides of one of the triangles as shown in Figure 2, so you can see that

The area of the square on the hypotenuse of a right-angled triangle is the sum of the areas of the squares on the other two sides.

That could be the title on your picture frame.

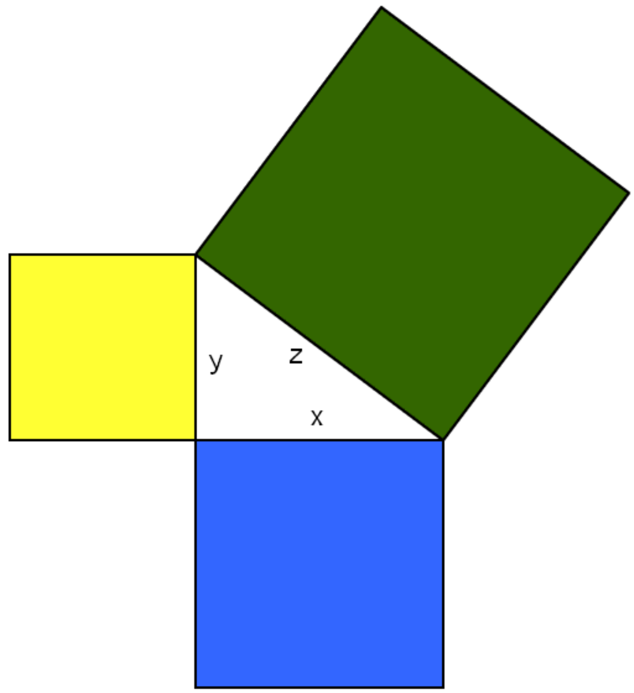

Figure 2: the squares on the sides of the triangle

Add the two smaller areas together (e.g. 9+16=25). Can the children recognise this square number and predict the hypotenuse? Does it agree with a careful measurement?

Finally, tie a knot to make a loop from a length of string or cord and mark it into 12 equal lengths.(Stretch the loop and make two folds and three equal doubled cords to mark six points and then subdivide each length.) Select three marks three, four and hence five intervals apart and stretch out the loop into a right-angled triangle using three sticks or pencils and a child or two. Build on this success as did 1the ancient Pyramid builders when your grandchildren build on Shelagh's triangles.

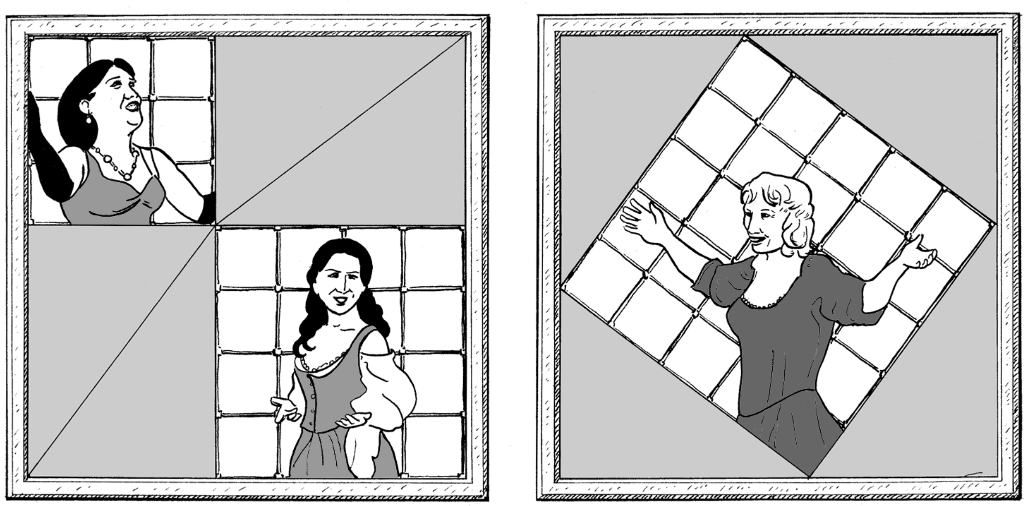

Figure 3: The Aria in the Hypotenuse is Equal to the sound of the Arias in the Other Two Sides

Figure 4: Pythagoras and his Granddaughters Decorate the House.

Pythagoras: "My dears, have you the right amount of paint? I have 25 cotylas of green for the large square."

Sophie: "We've got as much paint as you Grandpa."

I hope you (and any grandchildren) have enjoyed exploring the *Magic Picture Frame*. To explore more, try searching the internet for Shelagh's Triangles.